ATOMIC ENERGY CENTRAL SCHOOL-KAKRAPAR

CLASS- X

SUBJECT- MATHEMETICS

NAME OF THE CHAPTER- CIRCLE

MODULE-3

Subtopics-

Questions for practice.

Example 2 : Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Solution : We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact (see Fig. 10.9). We need to prove that

$$\angle PTQ = 2\angle OPQ$$

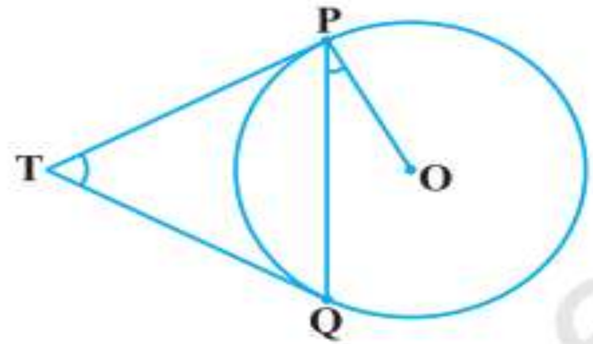


Fig. 10.9

Let $\angle PTQ = \theta$

Now, by Theorem 10.2, TP = TQ. So, TPQ is an isosceles triangle.

Therefore, $\angle TPQ = \angle TQP = \frac{1}{2}(180° - \theta) = 90° - \frac{1}{2}\theta$

Also, by Theorem 10.1, $\angle OPT = 90°$

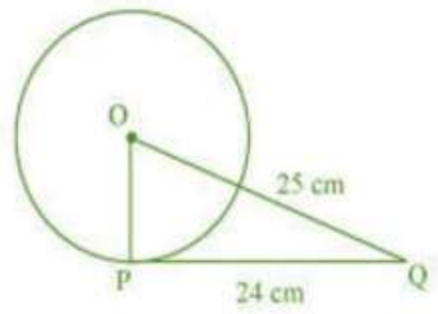
So, $\angle OPQ = \angle OPT - \angle TPQ = 90° - \left(90° - \frac{1}{2}\theta\right)$

$$= \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$$

This gives $\angle PTQ = 2\angle OPQ$

- **1.** From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(A) 7 cm (B) 12 cm(C) 15 cm (D) 24.5 cm

- Sol-



Let O be the centre of the circle.
Given: OQ = 25cm and PQ = 24 cm
We know that the radius is perpendicular to tangent. Therefore, $OP \perp PQ$
In ΔOPQ, By Pythagoras theorem,
$OP^2 + PQ^2 = OQ^2$
$\Rightarrow OP^2 + 24^2 = 25^2$ $\Rightarrow OP^2 = 625 - 576$ $\Rightarrow OP^2 = 49$ $\Rightarrow OP = 7$
Therefore, the radius of circle is 7 cm. Hence, the option (A) is correct.

In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that ∠ POQ = 110°,then ∠ PTQ is equal to

(A) 60°

(B) 70°

(C) 80°

(D) 90°

given that TQ and TP are two tnagent of the circle

we know that the radius is perpendicular to tangent,thus $OP \perp TP$ and $OQ \perp TQ$.

$\Rightarrow \angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$

In quadrilateral POQT, $\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$

Hence, the option (B) is correct.

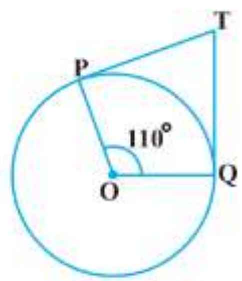
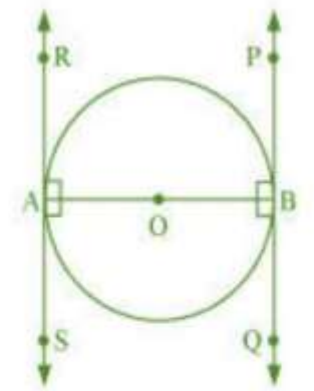


Fig. 10.11

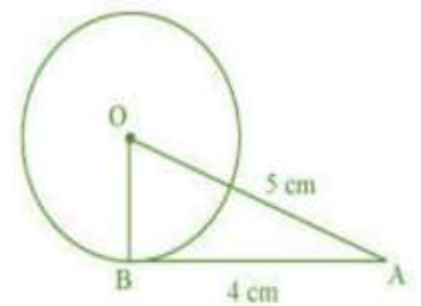
Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Let AB is diameter, PQ and RS are tangents drawn at ends of diameter.
We know that the radius is perpendicular to tangent. Therefore, $OA \perp RS$ and $OB \perp PQ$.
$\angle OAR = 90^\circ$ and $\angle OAS = 90^\circ$
$\angle OBP = 90^\circ$ and $\angle OBQ = 90^\circ$
From the above, we have
$\angle OAR = \angle OBQ$ [Alternate angles]
$\angle OAS = \angle OBP$ [Alternate angles]
Since, alternate angles are equal. Hence, PQ is parallel to PS.

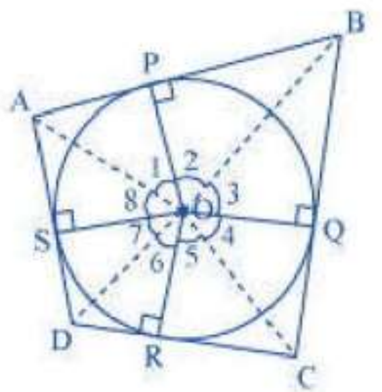
The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Let O be the centre and AB is a tangent at B.
Given: OA = 5cm and AB = 4 cm
We know that the radius is perpendicular to tangent.
Therefore, in ΔABO, $OB \perp AB$.
In ΔABO, by Pythagoras theorem,
$AB^2 + BO^2 = OA^2$
$\Rightarrow 4^2 + BO^2 = 5^2$
$\Rightarrow 16 + BO^2 = 25$
$\Rightarrow BO^2 = 9$
$\Rightarrow BO = 3$
Therefore, the radius of circle is 3 cm.



Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Quadrilateral ABCD is circumscribing a circle with centre O, touching at points P, Q, R and S.
Join the points P, Q, R and S from the centre O.
In ΔOAP and ΔOAS,



OP = OS	[Radii of same circle]
AP = AS	[Tangents drawn from point A]
AO = AO	[Common]
ΔOPA ≅ ΔOCA	[SSS Congruency rule]

Hence, $\angle POA = \angle SOA$ or $\angle 1 = \angle 8$
Similarly, $\angle 2 = \angle 3$, $\angle 4 = \angle 5$ and $\angle 6 = \angle 7$
Sum of all angles at point O is 360°. Therefore

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$
$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$\Rightarrow 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$
$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$
$\Rightarrow \angle AOB + \angle COD = 180°$
Similarly, we can prove $\angle BOC + \angle DOA = 180°$

Hence, the opposite sides subtend supplementary angles at the centre.

9. In Fig. 10.13, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

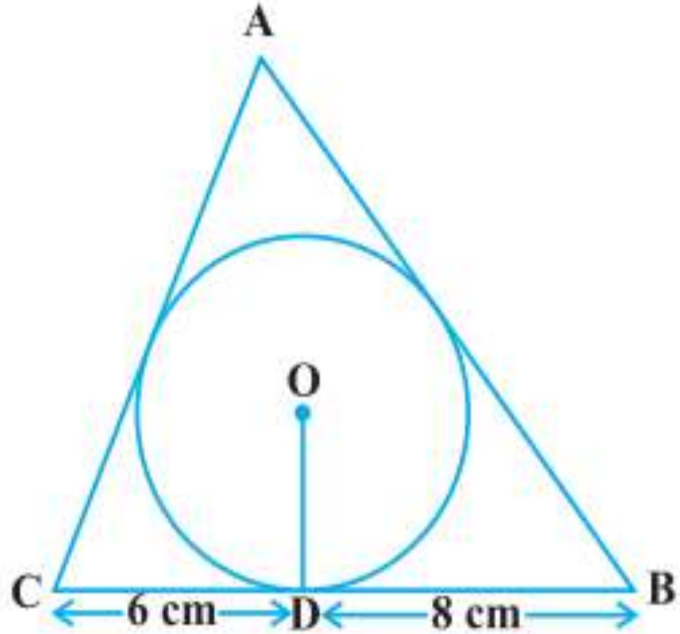


Fig. 10.14

THANK YOU